Under-Graduate Programme with MATHEMATICS; 1st Semester

MATHEMATICS | **MAJOR COURSE**

MMT126J: THEORY OF MATRICES | **Credits: 4 THEORY + 2 TUTORIAL**

Theory: 4 Credits (60 Hours) | **Tutorial: 2 Credits (30 Hours)**

Course Objectives: (i) To understand matrix theory as a tool to solve various problems of real life situations. (ii) to study the properties and applications of matrices.

Course Outcome: (i) After the completion of this course, students shall be able to use matrix as an operator to solve problems from various disciplines (ii) The matrix analysis can be used in coding theory, linear algebra and cryptography (iii) the stability of various systems can be established by using matrix analysis.

Theory: 4 Credits

Unit –I

Algebraic operations on matrices over R and C with emphasis on non-commutativity. Transpose A^T, inverse A^{-1}, and their basic properties including reversal laws. Conjugate transpose A^* and its algebraic properties. Structural properties of symmetric, skew-symmetric, Hermitian, and skew Hermitian matrices. Decomposition of a square matrix into the form $A = P + iQ$, where P is Hermitian and Q is skew-Hermitian. Basic idea of adjoint of a square matrix and its connection with matrix inversion.

Unit –II

Elementary row operations and their representation via elementary matrices. Row echelon form and reduction of a matrix to normal form PAQ. Invertibility of matrices and its connection with row reduction. Representation of non-singular matrices as products of elementary matrices. Systematic solution of linear homogeneous systems $AX = 0$ and non-homogeneous systems $AX = B$. Conditions for consistency and description of solution sets.

Unit –III

Linear combinations, span, and the concept of basis in R^n and C^n. Linear dependence and independence of vectors. Rank of a matrix via row-reduction and its interpretation as the number of linearly independent rows or columns. Relationship between rank r, number of variables n, and the dimension $n-r$ of the solution space of $AX = 0$. Trace of a matrix and its basic properties, including $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$.

Unit –IV

Matrix polynomials and algebraic functions of matrices. Characteristic polynomial and minimal polynomial of a matrix. Eigenvalues and eigenvectors as invariants of linear transformations. Cayley–Hamilton theorem and its use in computations. Inner product on R^n and C^n, norms, and orthogonality. Orthonormal sets and geometric interpretation of length and angles.

Tutorial: 2 Credits

Unit –V

Problems based Unit-I and Unit-II

Unit –VI

Problems based Unit-III and Unit-IV

Books Recommended:

1. A Text Book of Matrices, Aziz and Nisar, Kapoor Book Depot., Srinagar, 2015.
2. A textbook of Matrices, Shanti Narayan, S. Chand- 2019
3. Linear Algebra, K. Hoffman and R. Kunze, Pearson Education, 2nd edition-2015.
4. Linear Algebra, Schaum's outline series, Tata McGraw-Hill, 6th edition-2018.

Bachelors with Mathematics as Major
5th Semester
MMT522J1: Mathematics/Applied Mathematics: Algebra-I

Credits: 3 THEORY + 1 TUTORIAL **Theory: 45 Hours & Tutorial: 15 Hours**

Course Objective: To introduce students towards basic concepts of algebraic structures, viz groups and rings. (ii) to identify various properties associated with the groups and rings. (iii) to expose students towards advanced mathematics, such as advanced abstract algebra and communtative ring theory.

Course Outcomes: After the completion of this course, students shall be able to (i) understand group through symmetries in nature and can identify patterns. (ii) shall be able to apply these concepts in linear classical groups to solve problems arising in physics, computer science, economics and engineering etc.

Theory: 3 Credits

Unit I

Binary composition, equivalence relation and equivalence class. Groups: Definition and examples, Finite & Infinite groups, abelian & non-abelian groups. Properties of groups: uniqueness of identity/ inverse and cancellation laws. Subgroups and Cosets. Criterion of subgroups, order of an element, Lagrange's theorem. Cyclic groups: Definition & examples, cyclic groups are abelian, order of a cyclic group is equal to the order of its generator.

Unit II

Normal subgroups: Definition & examples, criterion of normal subgroups, product of subgroups, Quotient groups. Homomorphism, kernel of a homomorphism, Fundamental theorem of homomorphism, Isomorphism theorems (statements only), Normalizer of an element, Centre of a group. Permutation groups and Cayley's theorem. Definitions and examples of symmetric groups and alternating groups.

Unit III

Rings: Definition & Examples, elementary properties of rings, rings with & without zero divisors, Integral domains & skew fields. Definitions & examples of Fields, Subrings. Ideals. Quotient rings and Boolean rings. A finite integral domain is a field. Ring homomorphism, fundamental theorem of homomorphism & ring isomorphism. Prime, Maximal and Principal ideals (Definitions & examples only).

Tutorial: 1 Credit

Unit IV

Examples of monoids, semigroups, abelian/non-abelian, cyclic/non-cyclic groups, computing generators in a cyclic group, Structure theorem for cyclic groups (statement only), concept of even and odd permutation. Examples of commutative & non-commutative rings, Left & right ideal of a ring, relation between ideals & subrings, sum & product of ideals.

Recommended Books:

1. I. N. Herstein, Topics in Algebra, John Wiley, 1975.
2. Joseph Gallian , Abstract Algebra, Narosa Publishers, New Delhi, 1999.
3. M. Artin, Algebra, Pearson Education India, 2011.
4. D. S. Dumit and R. M. Foote, Abstract Algebra, John Wiley, 2003
5. P.B. Bhattachariya, S.K. Jain, S. R. Nagpaul, Basic Abstract Algebra, Cambridge University Press, 1994
6. Surjeet singh and Qazi Zameeruddin, Modern Algebra, S Chand And Company Ltd, 2021

Bachelors with Mathematics as Major

5th Semester

MMT522J2: Mathematics/Applied Mathematics:

MATHEMATICAL MODELING & NUMERICAL METHODS

Credits: 4 THEORY + 2 TUTORIAL **Theory: 60 Hours & Tutorial: 30 Hours**

Course Objectives: To understand role of mathematics in various disciplines, modeling real life situations. The objective of this course is to acquaint students with various analytical and numerical methods of finding solution of different type of problems, which arises in different branches of physics, chemistry, biology, humanities, social science etc.

Course Outcome: Students can handle physical and abstract problems to find an exact or approximated solutions. After getting trained a student can opt for advance courses in Applid Mathematics, Numerical analysis in higher mathematics.

Theory: 4 Credits

Unit- I

Introduction to Modeling, types of models, classification of mathematical models, flowchart of mathematical model, some basic models on simple pendulum, simple harmonic motion, projectile motion, gravitational laws. Population dynamics: exponential growth and logistic growth models. Lotka-Voltera population models.

Unit – II

Modeling heat conduction in solids, Fourier law of diffusion. Modeling heat transfer in biological tissues, fluids flow in human body, viscosity, classification of fluids, Poisselle's law, Simple mathematical models on chemical kinetics and law of mass action.

Unit – III

Rate of convergence, Errors: Relative, Absolute, Round off, Truncation. Numerical solution of nonlinear equations: Bisection method, Regula- Falsi method, Newton- Raphson method, Fixed-point Iteration method. Polynomial interpolation: Lagrange and Newtons divided difference interpolation, Central difference & averaging operators.

Unit – IV

Forward and backward difference interpolation, Hermite and Spline interpolation, piecewise polynomial interpolation. Numerical Integration: Some simple quadrature rules, , Trapezoidal rule, Simpsons $1/3^{rd}$ rule, Simpsons $3/8^{th}$ rule.

Tutorials: 2 Credits

Unit – V

Problems based on SHM, projectile motion, ecology and population dynamics. Problems on heat and mass transport in human body, problems on chemical kinetics.

Unit – VI

Problems on numerical approximation of algebraic and transcendental equations, some problems on Numerical differentiation and integration.

Recommended Books

1. J. N. Kapur, Mathematical Modeling, New Age International Publications, 2021.
2. MA Khanday. Introduction to Mathematical Modeling and Biomathematics, Dilpreet Publishers New Delhi, 2016.
3. J. N. Kapur, Mathematical methods in Biology and Medicine, Payal Book Distributor, 2008.
4. B. Bradie, A Friendly Introduction to Numerical Analysis, Pearson Education, India, 2007.
5. Kendall E. Atkinson: An Introduction to Numerical Analysis, 2008.
6. S. S. Sastry, Introductory method for Numerical Analysis, PHI New Delhi, 2012.

Bachelors with Mathematics as Major
5th Semester
MMT522J3: Mathematics/Applied Mathematics:
FOURIER AND LAPLACE TRANSFORM

Credits: 4 THEORY + 2 TUTORIAL **Theory: 60 Hours & Tutorial: 30 Hours**

Course Objectives: To develop skill in students about

i) Fourier series, Fourier and Laplace transforms as a tool to solve various problems of Mathematics.

ii) Integral transforms to be used in the field of applied Mathematics and especially in the field of physics and electronics to express periodic functions that comprise communication signal in waveform.

Course Outcome: After the completion of this course, students shall be able to use Fourier and Laplace transforms to solve the differential equations and to understand signal processing in frequency and time domain.

Theory: 4 Credi

Unit- I

Fourier Series: Introduction, Periodic functions: Properties, Even & Odd functions. Special waveforms: Square wave, Sawtooth wave, and Triangular wave. Euler's Formulae for Fourier Series, Fourier Series for functions of period 2π, Fourier Series for functions of period 2L, Dirichlet's conditions, Sum of Fourier series. If $f(x)$ is bounded and integrable function on $(-\pi,\pi)$ and if a_n, b_n are its Fourier coefficients, then $\sum a_n^2 + b_n^2$ converges. Half Range Series for sine and cosine functions, examples. Riemann Lebesque theorem.

Unit – II

The Fourier Transform. Periodic functions, Definition and examples of Fourier series, Drichlet's conditions, determination of Fourier coefficients, even and odd functions and their Fourier expansion, change of interval, half range series. Fourier transform, inverse Fourier transform, Fourier sine and cosine transforms and their inversion, properties of Fourier transforms Fourier transform of the derivative, convolution theorem, discrete Fourier transform and fast Fourier transform and their properties.

Unit – III

Definition, Laplace transform of elementary functions, Properties of Laplace transforms viz Linearity, translation, Change of Scale property etc. Laplace transform as periodic functions, Dirac-Delta function, Inverse Laplace transform. Laplace transform for derivatives, Laplace transform for integrals, Convolution theorem. Solution of ordinary differential equations with constant coefficients. Applications of partial differential equations.

Unit – IV

Application of Laplace transform to differential equation and integral equation. Application of Laplace transform to boundary value problems. Electrical circuits, dynamics, Beams, Heat conduction equations and wave equations.

Tutorials: 2 Credits

Unit – V

Problems based on unit-I and unit-II.

Unit – VI

Problems based on unit-III and unit-IV.

Recommended Books:

1. Ruel V.Churchill, Fourier Series & Boundary Value Problems, 8th Edition McGraw Hill Education 2011.
2. Davies, Brian, Integral Transforms and Their Applications, Springer, 2002.
3. Erwin, Kreysgiz, Advanced Engineering Mathematics, John Willey & Sons. 10th Edition, 2011.
4. K.S. Rao, Introduction to Partial Differential Equations, K.S. Rao, PHI, India.
5. Murrey R. Spiegel, Laplace Transforms, Schaum's outline series.
6. I. N. Sneddon: The use of Integral Transforms, McGraw-Hill, Singapore 1972.
7. R. R. Goldberg, Fourier Transforms, Cambridge University Press, 1961.
8. D. Brain, Integral Transforms and their applications, Springer, 2002.