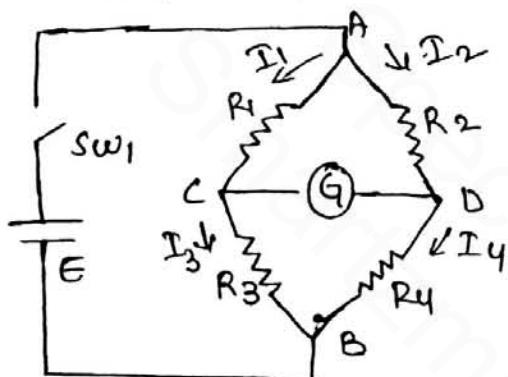




UNIT - IV

Bridges:-

A bridge that in its simplest form consists of a network of four resistance arms forming a closed loop with a dc source of current applied to two opposite junctions and a current detector connected to the other two junctions.



The bridge ckt's are extensively used for measuring component values such as R, L & C.

since the bridge ckt merely compares the value of unknown component with that of an accurately known component its measurement accuracy can be very high because the readout of this comparison is based on the null indication at bridge balance and is essentially independent of characteristics of the null detector.

∴ The measurement accuracy is therefore directly related to the accuracy of the bridge components.

and not to that of the null indicator used.

- Bridge ckt are employed to measure parameters R, F and D (Dissipation) of electronic circuits.
- DC bridges can measure resistance 'R' accurately over wide ranges.
- AC bridges can be used to determine the unknown values of inductor (L), capacitor (C) even resistance 'R' and also frequency 'f'.

Advantage with bridge measuring ckt's is that errors which occur in measurements due to parasitic values, temperature effects, errors due to improper grounding and shielding can be eliminated.

- The measurement range of the parameter is large. Even parameters like Q-factor of coil, D of a capacitor can also be measured using AC bridges.
- AC bridges are
 - 1) Maxwell
 - 2) Anderson
 - 3) Schering, Wien bridge
- DC bridges are
 - 1) Wheatstone & 2) Kelvin

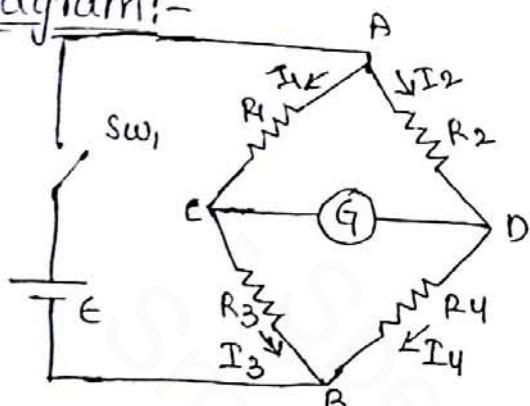


DC Bridges:-

what stone's Bridge (measurement of resistance):-

what stone's bridge is the most accurate method of resistance measurement.

Ckt diagram:-



EMF is connected to points A & B, also while the sensitive current indicating meter the galvanometer connected to points C & D.

- The Galvanometer is a sensitive micro ammeter with a zero centre scale. When there is no current through the meter the meter pointer rests at '0' (mid scale).
- Current in one direction causes the pointer to deflect one side and the current in other side.
- When the switch is closed current flows and divides in to the two arms at point A i.e. I_1 , I_2 .
Bridge is balanced when there is no current through the galvanometer i.e. the potential



difference at points C & D is equal.

To obtain the bridge balance equation we have from fig.

$$I_1 R_1 = I_2 R_2$$

for the galvanometer current to be zero, the following condition should be satisfied.

$$I_1 = I_3 = \frac{E}{R_1 + R_3}$$

$$I_2 = I_4 = \frac{E}{R_2 + R_4}$$

$$\frac{E R_1}{R_1 + R_3} = \frac{E R_2}{R_2 + R_4}$$

$$\Rightarrow \frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4}$$

$$R_1(R_2 + R_4) = R_2(R_1 + R_3)$$

$$R_1 R_2 + R_1 R_4 = R_1 R_2 + R_2 R_3$$

$$R_1 R_2 + R_1 R_4 - R_1 R_2 = R_2 R_3$$

$$R_4 = \frac{R_2 R_3}{R_1}$$

This is the equation for the bridge to be balanced in practical wheatstone bridge at least one of the resistors made adjustable; to permit balancing. When the bridge is balanced the unknown resistance may be determined from the setting of adjustable resistor i.e. std. resistor becoz it is precision device has small tolerance.



Hence

$$R_X = \frac{R_2 R_3}{R_1}$$

Ex from the above fig $R_1 = 10K$, $R_2 = 15K$, $R_3 = 40K$ find unknown resistance R_X

so:-

$$\begin{aligned} R_X &= \frac{R_2 R_3}{R_1} \\ &= \frac{15 \times 10^3 \times 40 \times 10^3}{10 \times 10^3} \end{aligned}$$

$$= 60 K\Omega$$

sensitivity of a whatsone bridge :-

when the bridge is in an unbalanced condition, current flows the galvanometer causing a deflection of its coil. The amount of deflection is function of sensitivity of galvanometer.

sensitivity:- deflection / unit current, if sensitivity is more meter deflects more amount for the same current. The deflection may be expressed in linear or angular units of measure so the sensitivity has units.

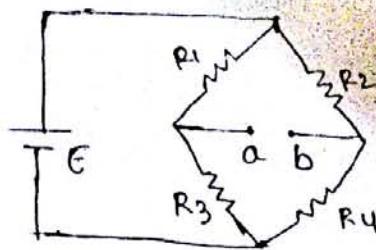
$$S = \text{mm}/\mu\text{A}$$

$$S = \text{degree}/\mu\text{A} \text{ or radians}/\mu\text{A}$$

\therefore The total deflection is $D = S \times I$

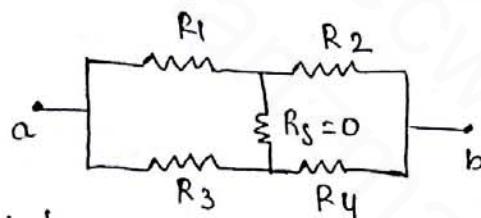
$$S = \text{sensitivity}$$

$$I = \text{current in } \mu\text{A}$$

unbalancedwhat is unbalance in bridge:-

To determine the amount of deflection results for a particular degree of unbalance. we must use thevenin's theorem. since we are interested in current through meter, we wish to find the thevenin's equivalent seen by the galvanometer.

R_{th} must be find from the ckt above.



$R_s = \text{internal resistance source}$

The thevenin's voltage is found by voltage divide equation. The voltage at point 'a' is given by

$$E_a = \frac{E \cdot R_3}{R_1 + R_3}$$

at point 'b'

$$E_b = \frac{E \cdot R_4}{R_2 + R_4}$$

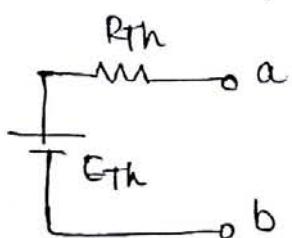
∴ voltage b/w a and b is ' $E_a - E_b$ '

$$E_{a-b} = E_a - E_b = \frac{ER_3}{R_1 + R_3} - \frac{ER_4}{R_2 + R_4}$$

$$E_{ab} = \left(\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} \right) E$$

To find R_{Th} replace 'E' by short ckt (its internal resistance).

$$R_{Th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4} \quad (R_1 || R_3 \text{ & } R_2 || R_4)$$

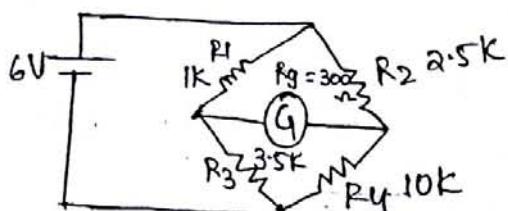


If the meter is connected b/w a & b, it will experience the same deflection, at the o/p of the bridge.

The magnitude of the current is limited by both the Thévenin's equivalent resistance and any resistance connected b/w a & b.

$$I_g = \frac{E_{Th}}{R_{Th} + R_g}$$

Ex an unbalanced Wheatstone bridge is given in fig b, calculate the current through the galvanometer.



Sol:-

$$E_{Th} = E_a - E_b = E_b - E_a$$

$$E_{Th} = \left(\frac{R_4}{R_2 + R_4} - \frac{R_3}{R_1 + R_3} \right) E$$



$$= 6 \left[\frac{10K}{2.5K+10K} - \frac{3.5K}{1K+3.5K} \right]$$

$$E_{Th} = 6(0.800 - 0.778)$$

$$E_{Th} = 0.132V$$

$$R_{Th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

$$R_{Th} = \frac{1K \times 3.5K}{1K + 3.5K} + \frac{2.5K \times 10K}{2.5K + 10K}$$

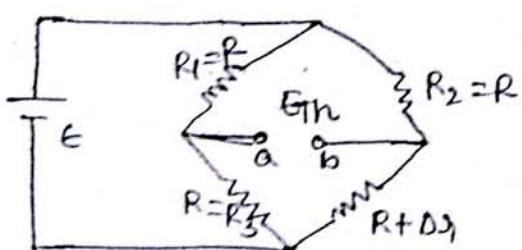
$$0.778K + 2K$$

$$= 2.778K$$

so $I_g = \frac{E_{Th}}{R_g + R_{Th}} = \frac{0.132V}{300 + 2.778K} = \frac{0.132}{3.078K} = 42.88 \mu A$

* slightly unbalanced wheat stone Bridge:-

If the three of the four resistors in a bridge equal to R and the fourth differs by $\pm 1\% (0.1\%)$ for thwinin's equivalent voltage and resistance I_{Th} shown below



$$E_a = \frac{ER}{R+R} = \frac{ER}{2R} = \frac{E}{2} //$$



$$E_b = \frac{E(R + \Delta r)}{R + R + \Delta r} = \frac{E(R + \Delta r)}{2R + \Delta r}$$

$$E_{Th} = E_a - E_b = \frac{E(R + \Delta r)}{2R + \Delta r} - \frac{E}{2} = E_b - E_a$$

$$= E \left[\frac{R + \Delta r}{2R + \Delta r} - \frac{1}{2} \right]$$

$$= E \left[\frac{-2R - \Delta r + 2(R + \Delta r)}{2(2R + \Delta r)} \right]$$

$$= E \left[\frac{-2R - \Delta r + 2R + 2\Delta r}{2(2R + \Delta r)} \right]$$

$$= E \left[\frac{\Delta r}{2(2R + \Delta r)} \right]$$

$$= E \frac{\Delta r}{4R + 2\Delta r}$$

If Δr is 5-10% less than Δr in the denominator can be neglected without introducing appreciable error. Therefore Thevenin's voltage is

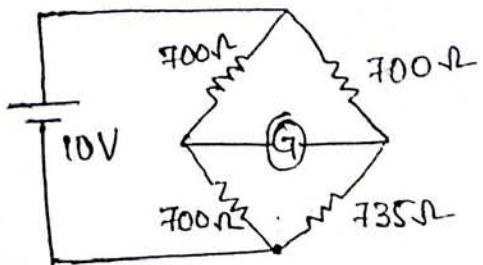
$$E_{Th} = \frac{E \times \Delta r}{4R} = E \left[\frac{\Delta r}{4R} \right]$$

$$\begin{aligned} R_{Th} &= \frac{R \times R}{R + R} + \frac{R(R + \Delta r)}{R + R + \Delta r} \\ &= \frac{R}{2} + \frac{R(R + \Delta r)}{2R + \Delta r} \end{aligned}$$

If Δr small compared to R , Δr can be neglected

$$R_{Th} = \frac{R}{2} + \frac{R}{2} = R$$

Ex Given a d'alembert 200-0-200μA movement having an internal resistance of 125Ω. calculate the current through the galvanometer given in fig by the approximation method.



Sol: Thevenin's equivalent voltage is -

$$\begin{aligned} E_{Th} &= \frac{E(\Delta R)}{4R} \\ &= \frac{10 \times 35}{4 \times 700} = 0.125 \text{ V} \end{aligned}$$

$$R_{Th} = R = 700 \Omega$$

$$I_g = \frac{E_{Th}}{R_{Th} + R_g} = \frac{0.125}{700 + 125} = \frac{0.125}{825} = 151.5 \mu\text{A}$$

If the detector is a 200-0-200μA galvanometer we see that the pointer

Applications of wheat stone bridge:-

- 1) To measure dc resistance various types of wires (Ex:- motor winding wires, relay coils & solenoids).
- 2) used to locate cable faults (BSNL) faults may be two line shorted single line grounded.



Limitations of Wheat Stone Bridge:-

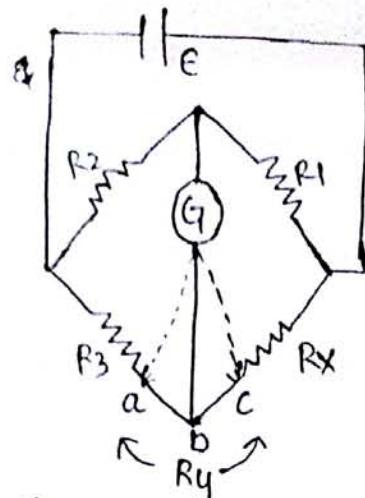
- 1) For low resistance measurements (resistance of leads contacts) it introduces an error.
- 2) For high resistance measurements, the resistance shown by the bridge is so high that the galvanic unit is less sensitive to imbalance, Battery replaced by DC supply & galvanometer replaced by VTVM. For M.R. wheat stone bridge can not be used.
- 3) Change in resistance of the bridge arms due to heating effect of current through the resistance, as the resistance to change and excess current may cause prominent change in the value.

Kelvin Bridge:-

Kelvin bridge is modified wheat stone bridge and used to measure the value of the resistance below:

1. In low resistance measurements, the resistance of leads connecting the unknown resistance to the terminals of the bridge circuit may affect the measurement.

Consider fig



' R_y ' represents the resistance of the connecting leads R_3 to R_x . The unknown resistance R_x can be connected either to point 'a' or to point 'c'.

- When the meter is connected to point 'a', the resistance ' R_y ' of the connecting lead is added to the unknown resistance R_x resulting in too high indication for R_x .
- When the meter is connected to point 'c', it is added to the arm R_3 and resulting measurement of ' R_x ' is lower than the actual value because R_3 is greater than the nominal value by the resistance R_y .
- If the galvanometer is connected to point 'b'. The ratio of resistance from 'c' to 'b' and that from a to b is the ratio of resistances $\frac{R_1}{R_2}$

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$

usual balance equation for the bridge give the relationship

$$(R_x + R_{cb}) = \frac{R_1}{R_2} (R_3 + R_{ab})$$

$$\therefore R_{ab} + R_{cb} = R_y \quad \text{and} \quad \frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$

$$= \frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1 + R_2}{R_2}$$

$$\Rightarrow \frac{R_{cb} + R_{ab}}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

$$\frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

$$\boxed{\therefore R_{ab} = \frac{R_y R_2}{R_1 + R_2}}$$

$$R_{cb} = R_y - R_{ab} = R_y - \frac{R_y R_2}{R_1 + R_2} \quad (\because R_{ab} + R_{cb} = R_y)$$

$$R_{cb} = \frac{R_1 R_y + R_2 R_y - R_2 R_y}{R_1 + R_2} = \frac{R_1 R_y}{R_1 + R_2}$$

$$\boxed{\therefore R_{cb} = \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1 R_y}{R_1 + R_2}}$$

Substitute R_{ab} , R_{cb} in equation $R_x + R_{cb} = \frac{R_1}{R_2} (R_3 + R_{ab})$

$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1}{R_2} \left(R_3 + \frac{R_2 R_y}{R_1 + R_2} \right)$$

$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_2 R_y}{R_2 (R_1 + R_2)}$$

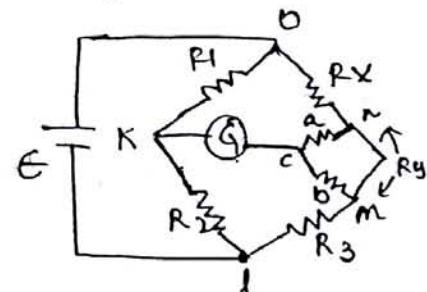
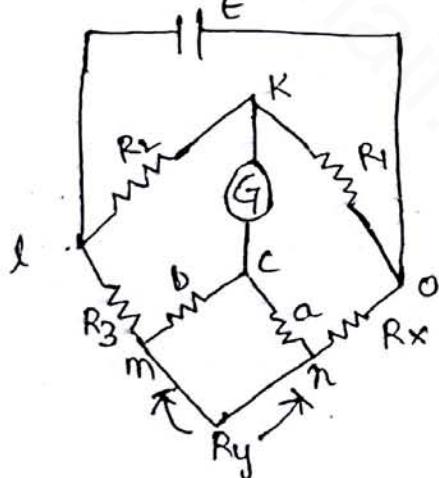


Hence $R_x = \frac{R_1 R_3}{R_2}$ (term to term equivalence)

This equation is usual balance what stone bridge an indicates that the effect of the resistance of the connecting leads. From point 'a' to point 'c' & and is eliminated by connecting the galvanometer to a position 'b'.

- This is the basic for the construction of Kelvins double bridge known as Kelvin bridge.
- It is a double bridge because it incorporates a second set of ratio arms.

Schematic diagram of Kelvin double bridge:-



The second set of arms 'a' & 'b' connects the Galvanometer to a point 'c' at the appropriate potential 'below m' connection 'i.e. Ry'.

- The ratio of the resistances of arms 'a' & 'b' is the same as the ratio of R_1 & R_2 the meter indication is zero when the potential at 'K' and are equal.

$$E_{IK} = E_{Imc}$$

$$\therefore E_{IK} = \frac{ER_2}{R_1+R_2}$$

$$E = I \left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} \right]$$

$$E_{IK} = I \left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} \right] \times \frac{R_2}{R_1+R_2}$$

similarly $E_{Imc} = I \left[R_3 + \frac{b}{a+b} \left[\frac{(a+b)R_y}{a+b+R_y} \right] \right]$

$$\text{But } E_{IK} = E_{Imc}$$

$$\frac{IR_2}{R_1+R_2} \left[R_3 + \frac{(a+b)R_y}{a+b+R_y} \right] = I \left[R_3 + \frac{b}{a+b} \left\{ \frac{(a+b)R_y}{a+b+R_y} \right\} \right]$$

$$R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \frac{R_1+R_2}{R_2} \left[R_3 + \frac{bR_y}{a+b+R_y} \right]$$

$$R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \frac{R_2 \left(\frac{R_1}{R_2} + 1 \right)}{R_2} \left(R_3 + \frac{bR_y}{a+b+R_y} \right)$$

$$R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \frac{R_1 R_3}{R_2} + \frac{R_3}{R_2(a+b+R_y)} + \frac{bR_1 R_y}{R_2(a+b+R_y)} + \frac{bR_y}{a+b+R_y}$$

$$R_x = \frac{R_1 R_3}{R_2} + \frac{bR_1 R_y}{R_2(a+b+R_y)} + \frac{bR_y}{a+b+R_y} - \frac{(a+b)R_y}{a+b+R_y}$$

$$= \frac{R_1 R_3}{R_2} + \frac{bR_1 R_y}{R_2(a+b+R_y)} + \frac{bR_y - aR_y - bR_y}{a+b+R_y}$$

$$= \frac{R_1 R_3}{R_2} + \frac{bR_1 R_y - R_2 a R_y}{R_2(a+b+R_y)}$$



$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_y}{a+b+R_y} \left[\frac{R_1}{R_2} - \frac{a}{b} \right]$$

But $\frac{R_1}{R_2} = \frac{a}{b}$

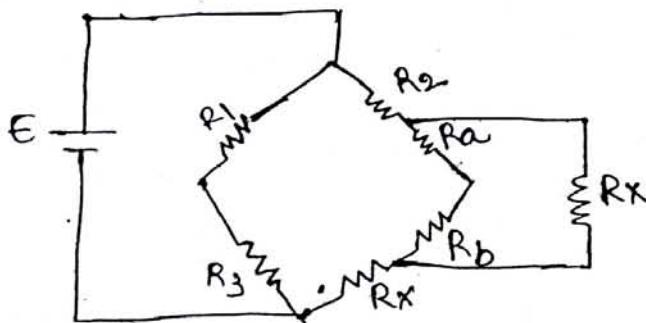
$$\therefore R_x = \frac{R_1 R_3}{R_2}$$

this is the usual equation for Kelvin's Bridge.

It indicates that the resistance of the connecting lead 'Ry' has no effect on the measurement, provided that the ratios of the resistances of the two sets of ratio arms are equal.

- In a typical Kelvin bridge the range of resistance covered is $1-0.00001\Omega$ ($10\text{M}\Omega$) with an accuracy of $\pm 0.05\%$ to $\pm 0.2\%$.

Ex: If in fig below the ratios of R_a & R_b is 200, R_1 is 5Ω and $R_1 = 0.5R_2$ what is the value of ' R_x '



$$\frac{R_1}{R_3} = \frac{a}{b}$$

$$R_2 R_3 = R_1 R_x$$

Sol:

$$\frac{R_x}{R_2} = \frac{R_b}{R_a}$$

$$\therefore \frac{R_x}{R_2} = \frac{R_b}{R_a} = \frac{1}{1000}$$

$$\frac{R_x}{R_2} = \frac{R_3}{R_1}$$

$$\frac{R_x}{R_2} = \frac{R_b}{R_a}$$

$$R_x = \frac{R_2 R_3}{R_1}$$

$$R_2 = \frac{R_1}{0.5} = \frac{5}{0.5} = 10\Omega$$

$$\frac{R_x}{10} = \frac{1}{1000}$$

$$R_x = \frac{10}{1000} = \frac{1}{100} = 0.01\Omega$$

Practical Kelvin double bridge:-

a commercial Kelvin bridge capable of measuring resistance from $10 - 0.00001\Omega$.

contact potential drops in the ckt may cause large errors. This effect is reduced by varying standard resistance consisting of nine steps of 0.001Ω each, plus calibrated manganin bar of 0.0011Ω with a sliding contact (R_3 total resistance $= 0.001 + 0.0011 = 0.002\Omega$).

- when both contacts are switched to select the suitable value of std resistance, the voltage drop b/w the rat arm connection points is changed, but the total resistor around the battery ckt is unchanged.

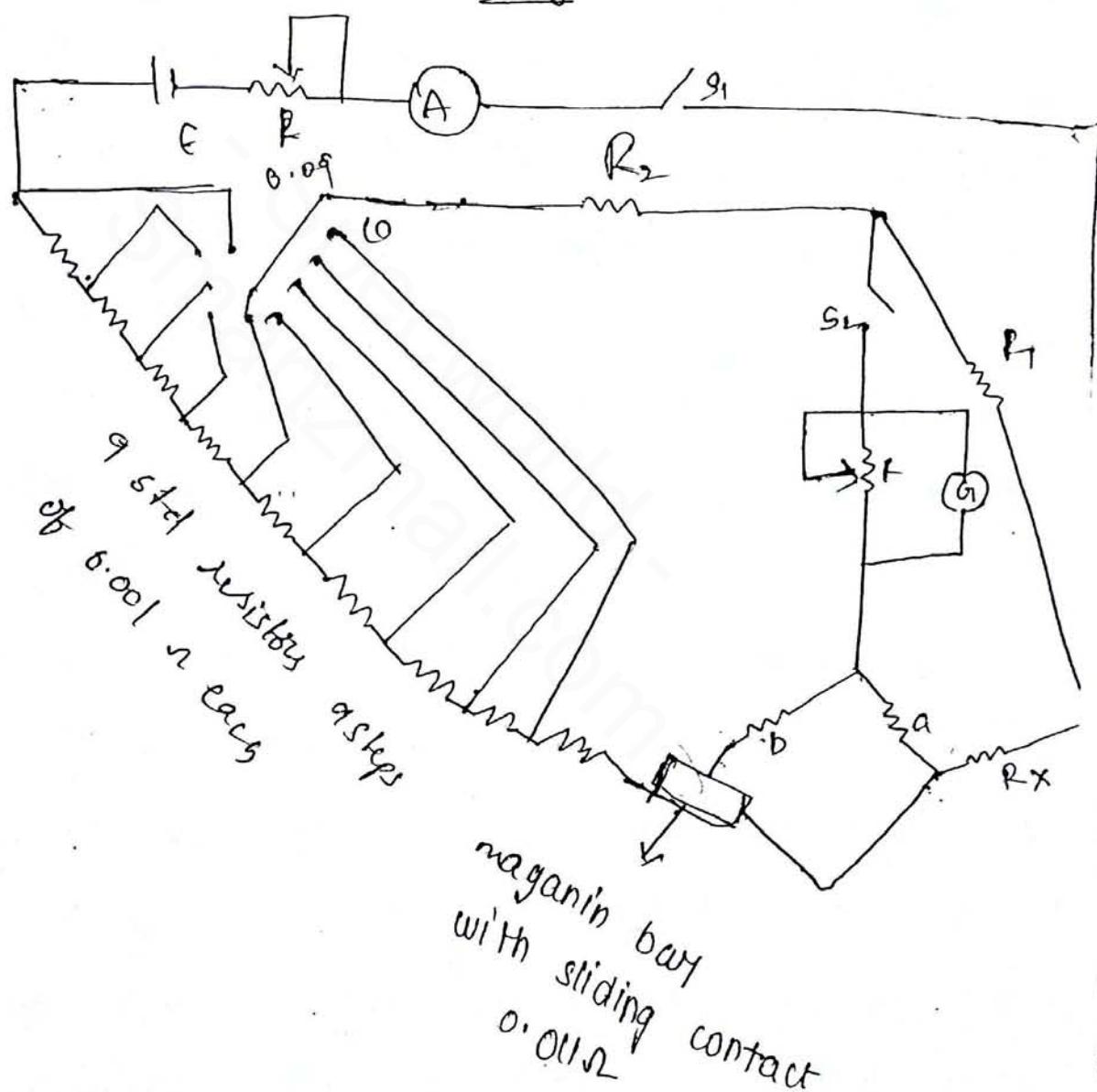
- This arrangement places any contact resistance in series with the relatively high resistance value of n arms, rendering the contact resistance effect negligible.

- The ratio $\frac{R_1}{R_2}$ is selected such that a relatively large part of the std resistance is used and h

R_x is determined to the largest possible number of significant figures.

- Therefore measurement accuracy improves, Kelvin bridge is shown below.

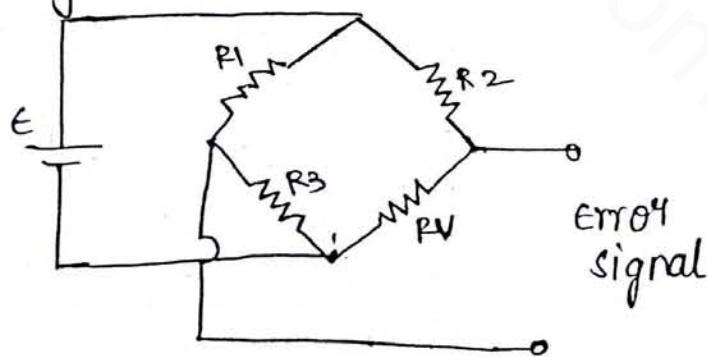
Practical Kelvin's double bridge:-





Bridge controlled ckt's:-

- when a bridge is unbalanced a potential difference exists at its o/p terminal and the current flows in the meter.
- when the bridge is used as an error detector in control ckt. The potential difference at the o/p of the bridge is called an error signal.
- passive ckt elements such as strain gauges, temperature sensitive resistors and photo resistors, produce no o/p voltage. However when used as one arm of what ston's bridge a change in their sensitive parameter produces a change in their resistances, the this causes the bridge to be unbalanced thereby producing an o/p voltage (or) an error signal.

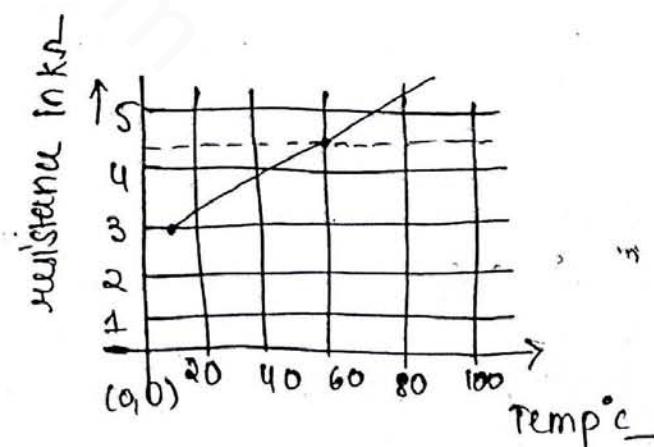
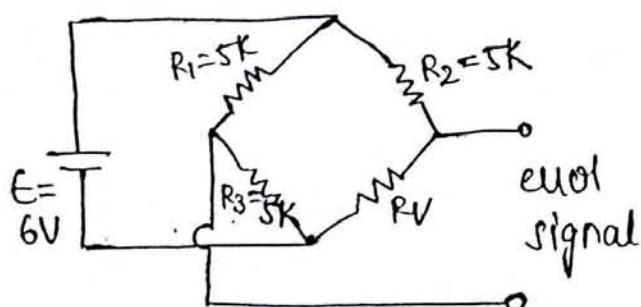


- R_V may be sensitive to one of the many different physical parameters such as heat (or) light.
- If the particular parameter to which the resistor is sensitive is of a magnitude such that the ratio $\frac{R_2 - R_1}{R_2}$, when the error signal is zero.



- If the physical parameter changes R_V also changes, the bridge then becomes unbalanced and an error signal occurs.
- In most control applications the measured parameter a controlled parameter is corrected, restoring R_V to the value that creates a null condition at the output of the bridge.
- Since R_V varies only by a small amount, the amplitude of the error signal is normally quite low. It is therefore amplified before being used for control purposes.

Ex: Resistor R_V in fig below is a temperature sensitive with a variation b/w resistance and temperature as shown in another fig below. calculate a) at what temporal the bridge is balanced and b) the amplitude of the error signal at 60°C .



Sol: a) The value of R_V when the bridge is balanced calculated as

$$R_V = \frac{R_2 R_3}{R_1} = \frac{5K \times 5K}{5K} = 5K$$



the bridge is balanced when the temperature is 80°C from the graph.

b) we can also determine resistance ' R_V ' at 60°C directly from the graph

$$\begin{aligned} e_s &= E \left[\frac{R_3}{R_1+R_3} - \frac{R_V}{R_2+R_V} \right] \\ &= 6 \left[\frac{5\text{K}}{5\text{K}+5\text{K}} - \frac{4.5\text{K}}{5\text{K}+4.5\text{K}} \right] \\ &= 6 (0.5 - 0.4736) = 6(0.0263) = 0.158\text{V} \end{aligned}$$

Output signal can also be determined by using the following equations.

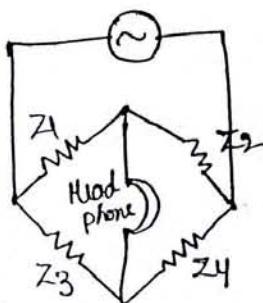
$$e_s = E_{th} = E \left[\frac{\Delta I}{4R} \right] = 6 \cdot \frac{500}{4 \times 5\text{K}} = \frac{3000}{20 \times 10^3} = 0.150\text{V}$$

$$e_s = 0.150\text{V}$$

AC Bridges:-

impedances at AF or RF are commonly determined means of an AC wheat stone bridge.

Diagram of an AC bridge



This is similar to dc bridge except the arms are



impedance except the bridge is energised by AC source and the galvanometer replaced by detector as headphones.

When the bridge is balanced

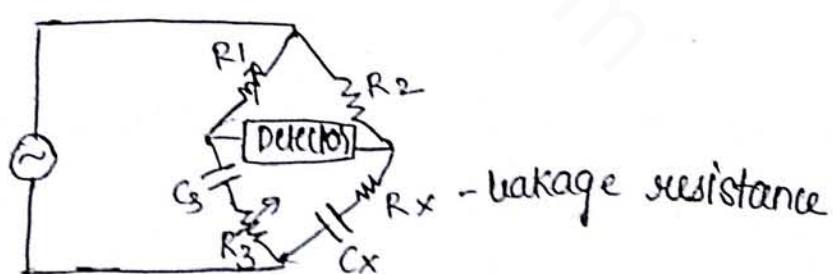
$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$

Z_1, Z_2, Z_3 , & Z_4 are impedances of the arms and are vector complex quantities that possess phase angles.

- It is thus necessary to adjust both the magnitude and phase angles of the impedance arms to achieve balance, i.e. the bridge must be balanced for both + reactance and the resistive component.

Capacitance comparison Bridge:-

CKT



The ratio arms R_1 and R_2 are resistive, the known C_3 is in series with the R_3 , R_3 is also a variable resistor used to balance the bridge, C_x is the unknown capacitor and R_x is the small leakage resistance of the capacitor.

- In this case an unknown capacitor is compared with

capacitor along with its leakage resistance. It is obtained by

$$Z_1 = R_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3 \text{ in series with } C_3 = Z_3 = R_3 - j/\omega C_3$$

$$Z_X = R_X \text{ in series with } C_X = Z_X = R_X - j/\omega C_X$$

The condition for the balance of the bridge is

$$Z_1 Z_X = Z_2 Z_3$$

$$R_1 \left[R_X - \frac{j}{\omega C_X} \right] = R_2 \left[R_3 - \frac{j}{\omega C_3} \right]$$

$$\therefore R_1 R_X - \frac{j R_1}{\omega C_X} = R_2 R_3 - \frac{j R_2}{\omega C_3}$$

TWO complex quantities are equal when both their real & their imaginary terms are equal. Therefore

$$R_1 R_X = R_2 R_3$$

$$\therefore R_X = \frac{R_2 R_3}{R_1}$$

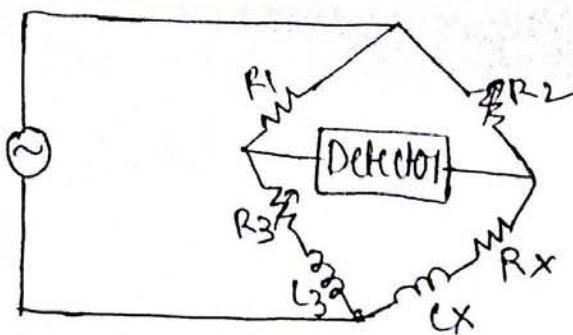
$$\frac{R_1}{\omega C_X} = \frac{R_2}{\omega C_3}$$

$$\therefore C_X = \frac{R_1 C_3}{R_2}$$

since R_3 does not appear in the expression for C_X a variable element, it is an obvious choice to eliminate any interaction b/w the two balance controls.

* inductance compensation bridge:-

CKT diagram:-



In this the unknown inductance 'L_x' and its internal resistors R_x are obtained by comparison with standard inductor and resistor L₃ & R₃.

The equation for the balance condition is

$$Z_1 Z_x = Z_2 Z_3$$

$$L_x = \frac{L_3 R_2}{R_1}$$

and the resistive balance equation yields

$$R_x = \frac{R_2 R_3}{R_1}$$

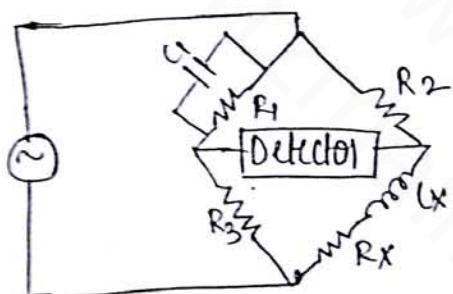
- In this bridge R₂ is chosen as inductive balance control and R₃ as resistance balance control.

- The balance is obtained alternatively varying L₃ or if the Q of the unknown resistance is greater than standard 'Q', it is necessary to place a variable resistor in series with the unknown resistance to obtain balance.
- If the 'L_x' has a high 'Q', it is permissible to vary the resistance ratio when a variable std inductor is not available.



Maxwell's Bridge:-

- Maxwell Bridge measures an unknown inductance in terms of a known capacitor the use of standard arm offers the advantage of say compactness and easy shielding.
- The capacitor is almost a loss less component, one a low resistance R_1 in parallel with C_1 and hence it is easy to write balance equation using the admittance of arm 1 instead of impedance.



The general equation for bridge balance is

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_X = \frac{Z_2 Z_3}{Z_1}$$

$Z_1 = R_1$ in parallel with C_1 i.e. $\frac{1}{Y_1} = \frac{1}{Z_1}$

$$Y_1 = \frac{1}{R_1} + j\omega C_1 \quad Z_3 = R_3$$

$Z_2 = R_2$; $Z_X = R_X$ in series with L_X

$$\boxed{Z_X = R_X + j\omega L_X}$$

$$R_X + j\omega L_X = \frac{Z_2 Z_3}{Z_1} = \frac{Z_2 Z_3}{1/Y_1} = Y_1 Z_2 Z_3$$

$$R_X + j\omega L_X = R_2 R_3 \left(\frac{1}{R_1} + j\omega C_1 \right)$$

$$R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + \frac{R_2 R_3 j\omega C_1}{1}$$

$$R_x = \frac{R_2 R_3}{R_1}$$

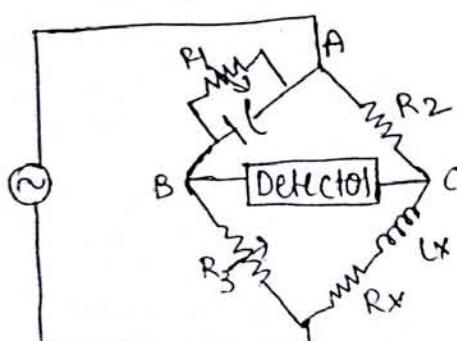
$$j\omega L_x = jR_2 R_3 C_1$$

$$L_x = R_2 R_3 C_1$$

Also $Q = \frac{\omega L_x}{R_x} = \frac{\omega C_1 R_2 R_3}{R_1} R_1 = \omega C_1 R_1$

- The maxwell bridge is limited to measurement of L_x values (1 to 10)
- The measurement is independent of excitation frequency. the scale of resistance can be calibrated directly to read inductance.
- The maxwell bridge using a fixed capacitor has the advantage that there is an interaction b/w the resistive and reactive balances. this can be avoided by varying the capacitors instead of R_2 & R_3 to obtain reactance. However the bridge can be made to read directly in 'Q'.

Maxwell Bridge (phasor diagram)

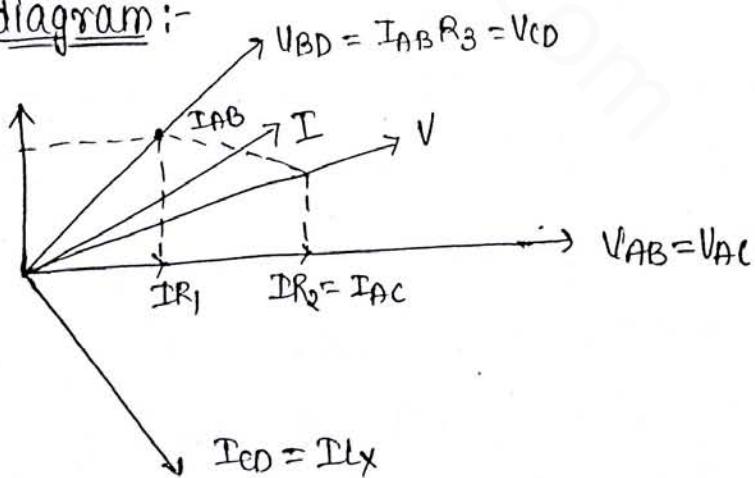




phasor:-

- 1) In arm AB R_1 , A_C are in parallel so take V_{AB} as reference voltage.
- 2) I_C leads V_{AB} by 90° (parallel). IR_1 in phase with V_{AB} . Vector summation of IR_1 & I_C is I_{AB} .
- 3) at balance $V_{AB} = V_{AC}$; R_2 is in arm AC, V_{AC} in phase I_{AC} .
- 4) at balance same I_{AB} flows through it by 90° in arm BC.
- 5) voltage across L_x lags the current through it by 90° . Current through R_x and VR_x are in phase. Therefore 3 is shown in fig.
- 6) total current I is vector summation of I_{AB} & I_{AC} .
- 7) total voltage V is vector summation of V_{AB} & V_{BD} .

phasor diagram:-



The Bridge is particularly suited for inductance measurement since, comparison with a capacitor is more ideal than with another inductance.

Commercial bridges measure 1-1000H with $\pm 2\%$ error.



Ex: A maxwell bridge is used to measure an inductance the bridge constants at balance are $C_1 = 0.01\text{uF}$, $R_1 = 470\text{k}\Omega$, $R_2 = 5.1\text{k}\Omega$, $R_3 = 100\text{k}\Omega$

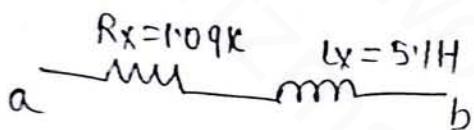
Q So: To find R_x & L_x

$$R_x = \frac{R_2 R_3}{R_1} = \frac{100\text{k} \times 5.1\text{k}}{470\text{k}} = 1.09\text{k}\Omega$$

$$L_x = R_2 R_3 C_1$$

$$= 5.1\text{k} \times 100\text{k} \times 0.01\text{uF} = 5.1\text{H}$$

The equivalent series circuit is shown below



Ex: The arms of an AC maxwell bridge are arranged as follows. AB and BC are non reactive resistors each 100Ω , DA a std variable reactor L_1 of resistance 32.7Ω and CD consists of std variable resistance R in series with a coil of unknown impedance Z . Balance was found with $L_1 = 50\text{mH}$ & $Z = 1.36R$ find the R & L of coil.

Q: Given, $R_1 = 32.7\Omega$, $I = 50\text{mA}$, $R_2 = 1.36\Omega$, $R_3 = 100\Omega$
To find ' r ' & ' L_2 ' where ' r ' is the resistance of the coil.

$$R_1 R_4 = R_3 (R_2 + r)$$

$$(32.7) \times 100 = 100 \times (1.36 + r)$$

$$100(32.7 - 1.36) = 100.91$$

$$\gamma = 32.7 - 1.36$$

$$\gamma = 31.34 \Omega //$$

2) To find L_2 :

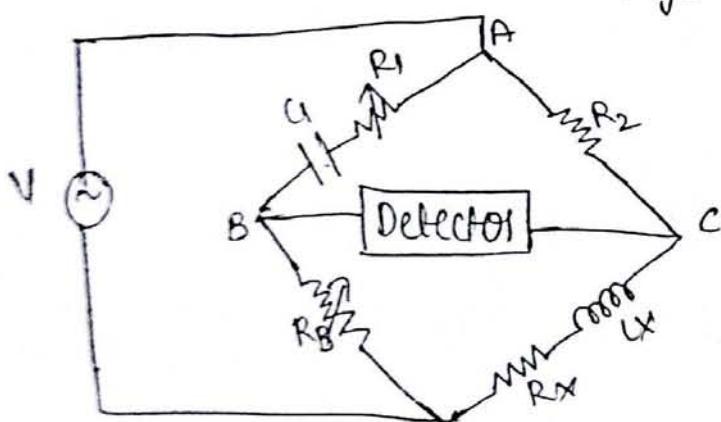
$$L_2 = L_1 \frac{R_4}{R_3} = 50 \text{ mA} \times \frac{100}{100}$$

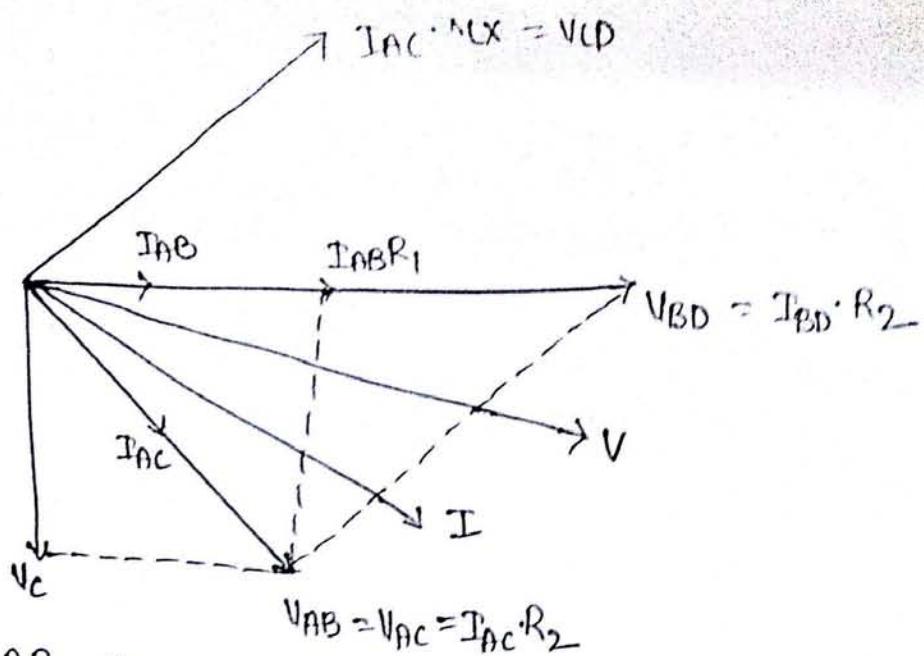
$$L_2 = 50 \text{ mA} //$$

Hays Bridge:-

Hays Bridge also used to measure unknown inductance in terms of known capacitance but arm 1 consists of capacitance in series with resistance unlike Maxwell bridge (instead of parallel).

- This Bridge is more convenient to measure inductance. High 'Q' coils for $Q=10$ the error is $\pm 1\%$. for $Q=30$ + error is $\pm 0.1\%$. so it is preferred for coils with high 'Q', and Maxwell's bridge for coils with low 'Q'.





- 1) In arm AB, the elements are in series therefore I_{AB} can be taken as reference. V_{C1} lags the current through C_1 by 90° . The vector sum of V_{C1} & $I_{AB}R_1$ is V_{AB} .
- 2) At balance $V_{AB} = V_{AC}$; I_{AC} & V_{AC} are in phase since there is the only element in arm 'AC'.
- 3) At balance I_{AB} , also flows through arm BD. R_B is the only element in arm BD.
- ∴ $I_{BD} = I_{AB}$ and V_{BD} will be in phase.
- 4) The drop across 'Lx' in arm 'CD' leads the current through L_x , I_{AC} by 90° as inductor is the element. Vector sum of $I_{AC}R_x$ and V_{CD} is V_{Lx} which is $V_{CD} = V_{BD}$.
- 5) Total current I is the vector sum of $I_{AB} = I_{AC}$. The vector sum of V_{AB} and V_{BD} is the total voltage.



at. balance

$$Z_1 Z_N = Z_2 Z_3$$

$$Z_1 = R_1 - j/\omega C_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_N = R_x + j\omega L_x$$

substitute in the balance equation

$$(R_1 - j/\omega C_1)(R_x + j\omega L_x) = R_2 R_3$$

$$R_1 R_x - \frac{j}{\omega C_1} R_x + j\omega L_x R_1 + \frac{L_x}{C_1} = R_2 R_3$$

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3$$

$$-\frac{R_x}{\omega C_1} + j\omega L_x R_1 = 0$$

$$\frac{R_x}{\omega C_1} = j\omega L_x R_1$$

$$R_x = \omega^2 C_1 R_1 L_x$$

substitute 'R_x' in eqn $R_1 R_x + \frac{L_x}{C_1} = R_2 R_3$

$$R_1 (\omega^2 C_1 R_1 L_x) + \frac{L_x}{C_1} = R_2 R_3$$

$$\omega^2 C_1^2 R_1^2 L_x + L_x = R_2 R_3 C_1$$

$$L_x (1 + \omega^2 C_1^2 R_1^2) = R_2 R_3 C_1$$

$$L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 C_1^2 R_1^2}$$

The 'ω' appears in both the terms R_x & L_x so the circuit is frequency sensitive.

i.e substituting L_x in $R_x = \omega^2 L_x C_1 R_1$

$$R_x = \frac{\omega^2 R_2 R_3 C_1}{1 + \omega^2 R_1^2 C_1^2} C_1 R_1$$

$$R_X = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1 + \omega^2 R_1^2 C_1^2}$$

The Hay's bridge is also used in the measurement of inductor inductance. The inductance balance equation depends on the losses of the inductor (α or Q) and also on the operating frequency.

The inconvenience given by this bridge is the balance condition for inductance, contains the multiplier $\frac{1}{1+1/Q^2}$. The inductance balance thus depends on its Q & frequency.

$$\therefore L_X = \frac{R_2 R_3 Q}{(1 + 1/Q^2)}$$

for a value of $Q > 10$ the term $1/Q^2$ will be small than $1/100$ and can be therefore neglected.

$$\therefore L_X = R_2 R_3 Q \quad \text{same as Maxwell equation}$$

commercial bridge measures from 1uH-100H with $\pm 2\%$
Ex Find the series equivalent inductance and resistance the N.W that causes an opposite angle to null with the following bridge arms.

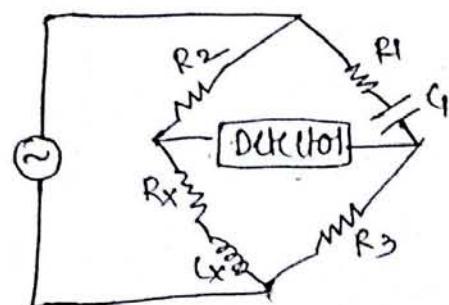
$$\omega = 3000 \text{ rad/sec}$$

$$R_2 = 10 \text{ k}\Omega$$

$$R_1 = 2 \text{ k}\Omega$$

$$C_1 = 1 \mu\text{F}$$

$$R_3 = 1 \text{ k}\Omega$$





Sol: we need to find R_x & L_x from equations

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1 + \omega^2 R_1^2 C_1^2} \quad \& \quad L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 R_1^2 C_1^2}$$

$$R_x = \frac{(3000)^2 \times 10K \times 2K \times 11C \times (1 \times 10^{-6})^2}{1 + (3000)^2 (2K)^2 (1 \times 10^{-6})^2}$$

$$= \frac{180 \times 10^3}{1+36} = \frac{180 \times 10^3}{37} = 4.86 \text{ k}\Omega$$

$$L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 R_1^2 C_1^2} = \frac{10K \times 11C \times (1 \times 10^{-6})}{1 + (3000)^2 (2K)^2 (1 \times 10^{-6})^2}$$

$$= \frac{10}{1+36} = \frac{10}{37} = 0.27 = 270 \times 10^{-3} \text{ H}$$

$$\therefore R_x = 4.86 \text{ k}\Omega, L_x = 270 \times 10^{-3} \text{ H}$$

Q2: Four arms of a Hay bridge are arranged as follows:
 AD is coil of unknown impedance Z , 'DC' is a non-inductive resistance of $1\text{k}\Omega$. CB is non inductive resistance of 800Ω in series with a std capacitor of $2\mu\text{F}$. BC is a non-inductive resistance of 16500Ω . If the supply frequency is 50Hz . calculate the value of L & R of coil when the bridge is balanced.

Sol: $R_2 = 1000\Omega, R_3 = 16500\Omega, R_4 = 800\Omega, C_4 = 2\mu\text{F}, f = 50\text{Hz}$

Step1: $\omega = 2\pi f = 2 \times 3.14 \times 50 = 314 \quad \& \quad \omega^2 = (314)^2 = 98596$

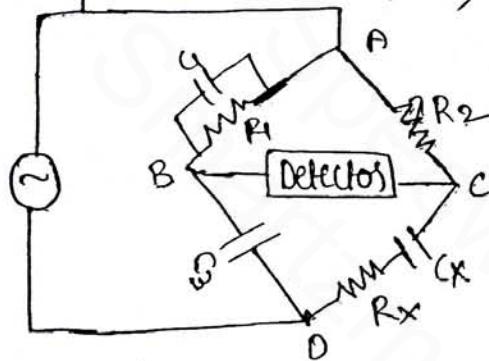
Step2: $L_x = \frac{R_2 R_3 C_4}{1 + \omega^2 C_4^2 R_4^2} \quad ; \quad R_x = \frac{\omega^2 C_4^2 R_4 R_2 R_3}{1 + \omega^2 C_4^2 R_4^2}$

$$\therefore I_1 = \frac{1000 \times 16500 \times 2 \times 10^6}{1 + 98596 \times (2 \times 10^6)^2 / (800)^2} = 26.417$$

$$R_1 = \frac{(314)^2 (2 \times 10^6)^2 (16500) (800) (1000)}{1 + (314)^2 (2 \times 10^6)^2 (800)^2} = 4.18 \text{ k}\Omega$$

SCHERING BRIDGE:-

This used to measure the capacitance of capacitor and insulating properties (dielectric)



C_3 is high quality mica capacitor (low loss) for general measurement.

(b) Air capacitor for insulation measurements.

For balance, the general equation

$$Z_1 Z_X = Z_2 Z_3$$

$$Z_X = \frac{Z_2 Z_3}{Z_1} ; \quad Z_X = Z_2 Z_3 Y_1$$

$$Z_X = R_X - j/\omega C_X$$

$$Z_2 = R_2$$

$$Z_3 = -j/\omega C_3$$

$$Y_1 = 1/R_1 + j\omega C_1$$

$$\boxed{Z_X = Z_2 Z_3 Y_1}$$

$$R_X - j/\omega C_X = R_2 (-j/\omega C_3) \left(\frac{1}{R_1} + j\omega C_1 \right)$$

$$\cdot \left(R_x - \frac{1}{\omega C_x} \right) = \frac{R_2(-j)}{R_1 W C_3} + \frac{R_2 C_1}{C_3}$$

equating real and imaginary terms

$$R_x = \frac{R_2 C_1}{C_3}$$

$$C_x = \frac{R_1 C_3}{R_2}$$

The dual of capacitor C_1 can be calibrated directly to give the dissipation factor at a particular frequency.

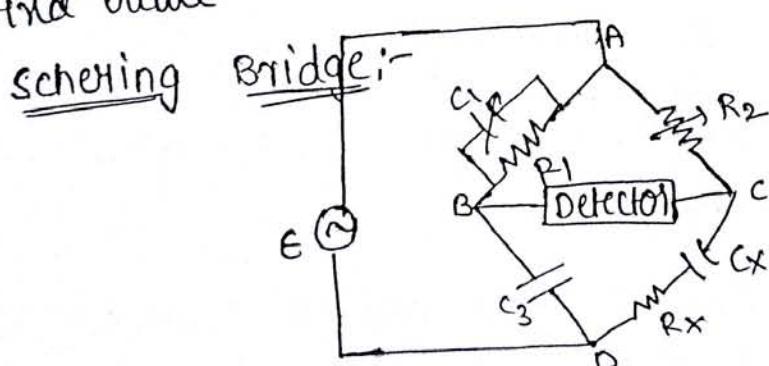
- The dissipation factor D of a series RC ckt is defined as the cotangent of the phase angle

$$D = \frac{R_x}{X_x} = \omega C_x R_x$$

D is reciprocal of quality factor ' α ' ($D=1/\alpha$) indicates the quality of the capacitor.

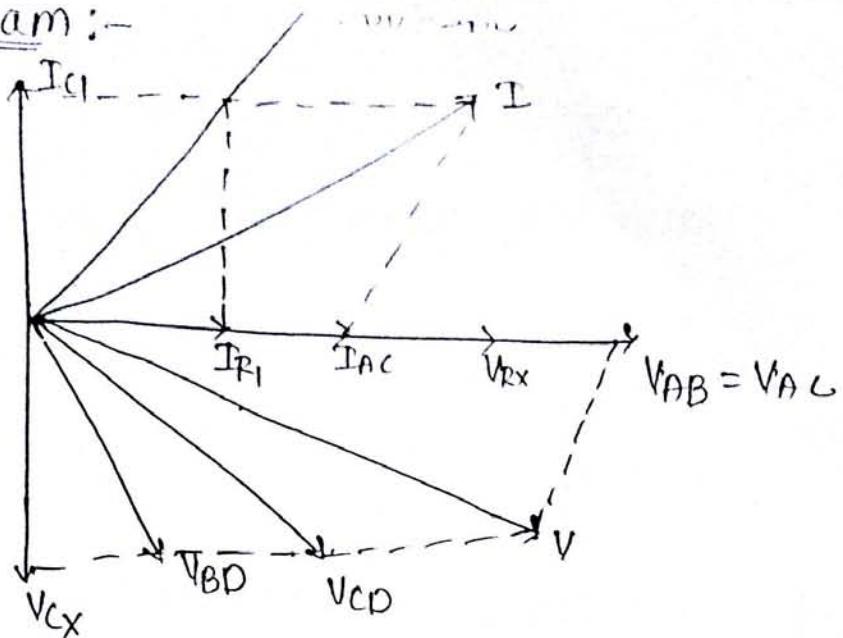
→ commercial schuring bridge measures from 100pf-1μF & $f \pm 2\%$ accuracy.

→ The dual of C_3 is gradual in terms of direct reading capacitor C_x . If the resistance ratio is maintained at a fixed value.





Phasor diagram:-



- 1) In arm AB R_1 & C_1 are in parallel therefore V_{AB} is taken as reference.
 - 2) At balance $I_{AB} = I_{BD}$, V_{BD} lags with respect to I_{BD} .
 - 3) At balance $V_{AB} = V_{AC}$. I_{AC} will be in phase with V_{AC} .
 - 4) At balance $I_{AC} = I_{CD}$: I_{AC} and V_{RX} will be in phase. V_{CX} lags I_{CX} by 90° .
 - 5) Total voltage V is the vector sum of ' $V_{AB} = V_{BD}$ '
 - 6) Total current I is vector sum $I_{AB} + I_{AC}$.
- The bridge is widely used for testing small capacitors at low voltages with very high precision.
 - If the lower junction of bridge is grounded if reactance of C_1 and C_2 are much higher than the resist of R_1 & R_2 . Hence most of the voltage across C_2 & C_1 is very little across R_1 & R_2 .
 - If the junction of R_1 & R_2 is grounded the detector is effectively at ground potential, this reduces any stray capacitance effect & makes the bridge more stable.

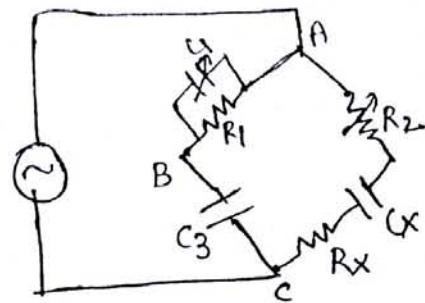
Ex 1 An ac bridge has the following constants

$$\text{arm AB} = 0.15\mu\text{f} \parallel 1\text{k}\Omega$$

$$\text{arm AD} = 2\text{k}\Omega$$

$$\text{arm BC} = 0.15\mu\text{f}$$

$$\text{arm CD} = C_x \text{ series with } R_x$$



frequency - 11CHZ

$$\text{so: from eqn's } R_x = \frac{R_2 C_1}{C_3}, \quad C_x = \frac{R_1}{R_2} C_3$$

$$R_x = \frac{0.15\mu\text{f}}{0.15\mu\text{f}} \times 2\text{k} = 2\text{k}\Omega$$

$$C_x = \frac{R_1}{R_2} C_3 = \frac{1\text{k}}{2\text{k}} \times 0.15\mu\text{f} = 0.25\mu\text{f}$$

Dissipation factor is given by

$$D = \omega C_x R_x = 2 \times 3.1416 \times 1000 \times 2\text{k} \times 0.25\mu\text{f}$$

$$= 3.1416$$

Ex 2 A sheet of 4.5mm thick Bakelite is tested at 50Hz
12cm in diameter the schering bridge uses a std air
capacitor C_2 of $105\mu\text{f}$ capacitor, a non reactive, R_4 of $\frac{100}{\pi}$
parallel with a variable capacitor and is obtained with
 $\omega = 0.5\mu\text{f}$ and $R_3 = 260\Omega$ calculate the capacitance, P
and relative permittivity of the sheet.

$$\text{Sol: } d = \text{Thickness of the sheet in mm} = 4.5 \times 10^{-3} \text{ m}$$

$$f = 50\text{Hz}$$

$$\omega = 2\pi f = 2 \times 3.14 \times 50 = 314$$

$$A = \text{area of electrodes in meters}^2 = \pi (6 \times 10^{-2})^2$$

$$C_2 = 105 \times 10^{-12}, \quad R_4 = \frac{1000}{37\pi}, \quad C_4 = 0.5\mu\text{f}, \quad R_3 = 260\Omega$$

Step 1: $Q_1 = \frac{R_4}{R_3} \times C_2 = \frac{1000}{71 \times 260} \times 1050 \times 10^{-12} = 128.7 \text{ pF}$

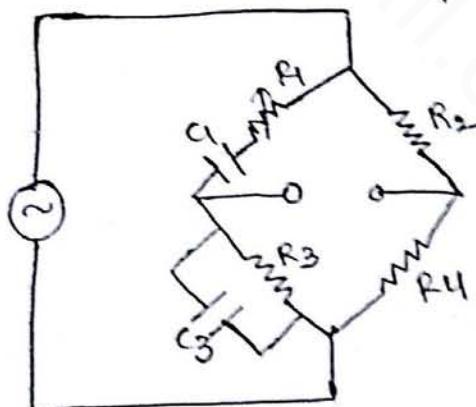
Step 2: PF : $\omega R_4 C_4 = 2 \times 3.14 \times 50 \times \frac{1000}{\pi} \times 0.5 \times 10^{-6} = 0.05$

Step 3: capacitance is given by $Q = \epsilon_r \epsilon_0 \frac{A}{d}$

$$\epsilon_r = \frac{Qd}{\epsilon_0 A} = \frac{128.7 \times 10^{-12} \times 4.5 \times 10^{-3}}{8.854 \times 10^{-12} \times \pi \times (6 \times 10^{-2})^2} = 5.786$$

WEN BRIDGE:-

- wein bridge has series RC combination in one arm parallel RC combination in adjoint arm.
- wein bridge is basically designed to measure freq if it can also be used for the measurement of an unknown capacitor with great accuracy.



The impedance of one arm is $Z_1 = R_1 - j\omega C_3$

The admittance of parallel arm is $Y_3 = \frac{1}{R_3} + j\omega C_3$

using the bridge balance equation

$$Z_1 Z_4 = Z_2 Z_3$$

$$\therefore Z_1 Z_4 = \frac{Z_2}{Y_3}$$

$$Z_2 = Y_3 Z_1 Z_4$$



$$R_2 = R_4 \left[R_1 - \frac{j}{\omega C_1} \right] \left\{ \frac{1}{R_3} + j\omega C_3 \right\}$$

$$R_2 = R_4 \left[\frac{R_1}{R_3} + jR_1\omega C_3 - \frac{j}{\omega C_1 R_3} + \frac{\omega C_3}{\omega C_1} \right]$$

$$R_2 = \frac{R_1 R_4}{R_3} + jR_4 R_1 \omega C_3 - \frac{j R_4}{\omega C_1 R_3} + \frac{R_4 C_3}{C_1}$$

$$R_2 = \left(\frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1} \right) - j \left(\frac{R_4}{\omega C_1 R_3} - R_4 R_1 \omega C_3 \right)$$

equating real and imaginary terms we have

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1}$$

$$\frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4 = 0$$

$$\therefore \boxed{\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}}$$

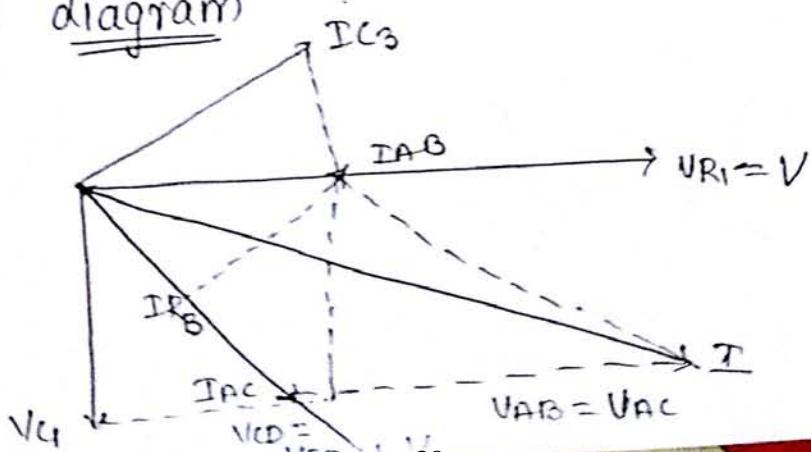
$$\& \quad \frac{R_4}{\omega C_1 R_3} = \omega C_3 R_1 R_4 \Rightarrow \frac{1}{\omega C_1 R_3} = \omega C_3 R_4$$

$$\omega^2 = \frac{1}{C_1 C_3 R_3 R_1}$$

$$\omega = \frac{1}{\sqrt{C_1 C_3 R_3 R_1}}$$

$$\omega = 2\pi f \quad \therefore f = \frac{1}{2\pi\sqrt{C_1 C_3 R_3 R_1}}$$

phasor diagram





The conditions for bridge balance $\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}$ & determine required resistance ratio for balance.

$$f = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}} \text{ determines the freq of IIP 0}$$

If we satisfy equation $\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}$ and excite the

bridge with frequency 'f' the bridge will be balanced.

- In most win bridge ckt's the components are chosen

such that $R_1 = R_3 = R$, $C_1 = C_3 = C$ eqn $\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}$ reduces to '2' and $f = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}}$ reduces to $f = \frac{1}{2\pi R C}$

which is general equation for the frequency of the bridge ckt.

- The bridge is used for measuring frequency in the audio range. resistance R_1 & R_3 can be ganged together to have identical values. capacitor C_1 & C_3 are normally of fixed values.

- The audio range is normally divided into 20-200-2K ranges. In this case the resistance can be used for range changing and the capacitor 'C' & 'C3' for the frequency control.

- The Bridge can also be used for measuring capacitors. In this case frequency of operation must be known.

- The bridge is also used in a harmonic distortion analyzer, as a notch filter, and in audio frequency



and radio frequency oscillators as a frequency oscillator as a frequency determining element.

- An accuracy of 0.5% - 1% can be obtained using this b because its frequency sensitive, it is difficult to balance unless the waveform of applied voltage is purely sinus

Ex: A Wien bridge ckt consists of the following

$$R_1 = 4.7 \text{ k}\Omega \quad C_1 = 5 \text{ nF}$$

$$R_2 = 20 \text{ k}\Omega \quad C_3 = 10 \text{ nF}$$

$$R_3 = 10 \text{ k}\Omega$$

$$R_4 = 100 \text{ k}\Omega$$

determine the frequency of the circuit.

$$\text{Sol: } f = \frac{1}{2\pi \sqrt{C_1 C_3 R_1 R_3}}$$

$$f = \frac{1}{2\pi \sqrt{5 \times 10^{-9} \times 10 \times 10^{-9} \times 4.7 \times 10^3 \times 10 \times 10^3}}$$

$$= \frac{1}{2\pi \sqrt{5 \times 10^{-10} \times 4.7}}$$

$$f = \frac{10^5}{2\pi \sqrt{5 \times 4.7}} = 3.283 \text{ kHz}$$

Ex: An ac bridge with terminals ABCD has in arm A a resistance of 800Ω in parallel with a capacitor of $5 \mu\text{F}$, arm BC - a resistance of 4000Ω in series with a capacitor of $1 \mu\text{F}$, arm CD - resistance of 1000Ω arm DA - a pure resistance R .

a) determine the value of frequency for which the bridge is balanced.

b) calculate the value of R required to produce balance.

Sol: The bridge configuration of Wien bridge

Given, $G = 0.5 \text{ mho}$, $R_1 = 800 \Omega$

$C_3 = 1.0 \mu\text{F}$, $R_3 = 400 \Omega$

$R_4 = 1000 \Omega$, $R_2 = R = ?$

Step 1: Frequency calculated by

$$\begin{aligned} f &= \frac{1}{2\pi \sqrt{R_1 G_1 R_3 C_3}} \\ &= \frac{1}{2\pi \sqrt{800 \times 0.5 \times 10^{-6} \times 400 \times 1 \times 10^{-6}}} \\ &= \frac{1}{2\pi \sqrt{800 \times 400 \times 0.5 \times 10^{-12}}} \\ &= \frac{10^6}{2\pi \sqrt{800 \times 200}} \\ &= \frac{10^6}{2\pi \times 400} = \frac{1000 \text{ kHz}}{2 \times 3.14 \times 400} = 0.398 \text{ kHz} \end{aligned}$$

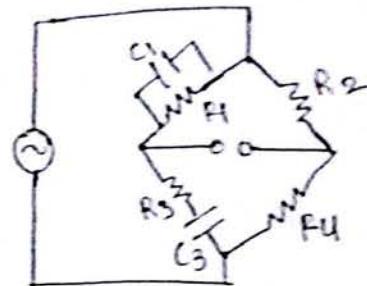
Step 2: also given

$$\frac{R_2}{R_1} + \frac{G_1}{C_3} = \frac{R_4}{R_3}$$

$$\frac{400}{800} + \frac{0.5 \times 10^{-6}}{1 \times 10^{-6}} = \frac{1000}{R}$$

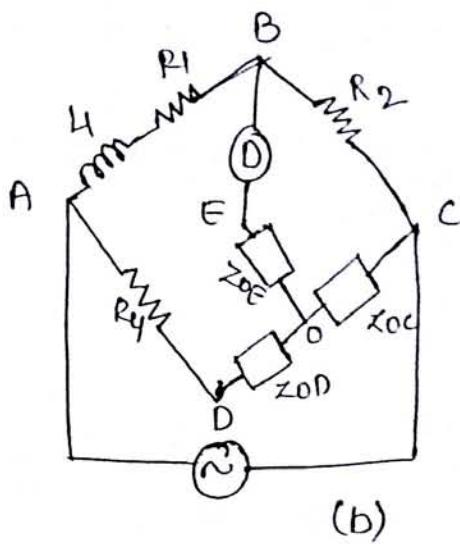
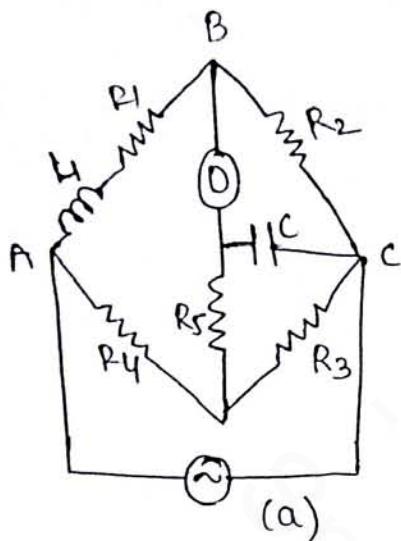
$$0.5 + 0.5 = \frac{1000}{R}$$

$$R = 1000 \Omega$$



ANDERSON BRIDGE:-

The Anderson's Bridge is modification of Maxwell-Wien bridge shown below



The balance condition for this bridge can be easily obtained converting the mesh impedances Z_1, Z_3, Z_4 to an equivalent star point O as shown in fig (b).

As per delta to star

$$Z_{OD} = \frac{R_3 R_5}{R_3 + R_5 + 1/j\omega c} ; \quad Z_{OC} = \frac{R_3 / j\omega c}{R_3 + R_5 + 1/j\omega c}$$

Hence with ref to fig(b) it can be seen that

$$Z_1 = (R_1 + j\omega L_1) ; \quad Z_2 = R_2 ; \quad Z_4 = R_4 + Z_{OD} ; \quad Z_3 = Z_{OE}$$

$$Z_{OC} = \frac{R_3 / j\omega c}{R_3 + R_5 + 1/j\omega c}$$

for balance condition

$$Z_1 Z_3 = Z_2 Z_4$$

$$\therefore (R_1 + j\omega L_1) \cdot Z_{OC} = Z_2 (Z_4 + Z_{OD})$$

$$(R_1 + j\omega L_1) \left(\frac{R_3 / j\omega c}{R_3 + R_5 + 1/j\omega c} \right) = R_2 \left[R_4 + \frac{R_3 R_5}{R_3 + R_5 + 1/j\omega c} \right]$$



$$\frac{(R_1 + j\omega L_1) \frac{R_3/j\omega C}{(R_3 + R_5 + \cancel{j\omega C})}}{(R_3 + R_5 + \cancel{j\omega C})} = \frac{R_2 (R_4 (R_3 + R_5 + \frac{1}{j\omega C}) + R_3 R_5)}{(R_3 + R_5 + \cancel{j\omega C})}$$

$$(R_1 + j\omega L_1) \left(\frac{R_3}{j\omega} \right) = R_2 R_4 R_3 + R_2 R_4 R_5 + \frac{R_2 R_4}{j\omega C} + R_2 R_3 R_5$$

$$\frac{R_1 R_3}{j\omega C} + \frac{R_3 j\omega L_1}{j\omega C} = R_2 R_3 R_4 + R_2 R_4 R_5 + \frac{R_2 R_4}{j\omega C} + R_2 R_3 R_5$$

$$-\frac{jR_1 R_3}{\omega C} + \frac{R_3 L_1}{C} = R_2 R_3 R_4 + R_2 R_4 R_5 - \frac{jR_2 R_4}{\omega C} + R_2 R_3 R_5$$

equating real & imaginary

$$\frac{L_1 R_3}{C} = R_2 R_3 R_4 + R_2 R_4 R_5 + R_2 R_3 R_5$$

$$L_1 = \frac{C}{R_3} [R_2 R_3 R_4 + R_2 R_4 R_5 + R_2 R_3 R_5]$$

$$L_1 = CR_2 \left(R_4 + \frac{R_4 R_5 + R_5}{R_3} \right)$$

$$-\frac{jR_1 R_3}{\omega C} = -\frac{jR_2 R_4}{\omega C}$$

$$\therefore R_1 = \frac{R_2 R_4}{R_3}$$

This method is capable of precise measurement of inductances and a wide range of values from a few uH to several Henrys.

Ex: An inductive coil was tested by an Anderson bridge. The following were the values on bridge.

AB = unknown R_1 in series with unknown L_1

BC = 1000Ω , CD = 1000Ω , DA = 2000Ω respectively

A capacitor of 10mF and resistance 400Ω are



connected b/w C & D respectively, source b/w A & C
 $\gamma = 496$, determine L & R.

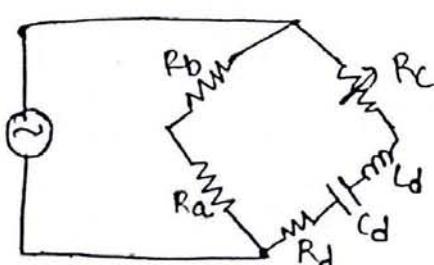
So:- Given $R_2 = 200\Omega$, $R_3 = 1000\Omega$, $R_4 = 1000\Omega$, $C = 10\mu F$,
 $\gamma = 496$

Step 1: To calculate $R_1 = \frac{R_2 R_3}{R_4} = \frac{200 \times 1000}{1000} = 200\Omega$

Step 2 To calculate

$$\begin{aligned} L &= \frac{CR_3}{R_4} (\gamma R_4 + R_2 R_4 + \gamma R_2) \\ &= \frac{10 \times 10^{-6} \times 1000}{1000} (496 \times 10^3 + 200 \times 10^3 + 496 \times 200) \\ &= 10^3 \times 10^3 (496 + 200 + 0.496 \times 200) \\ &= 10^2 (496 + 200 + 99.2) \\ &= 795.2 \times 10^{-2} \\ &= 7.952 H \end{aligned}$$

Resonance Bridge:-



one arm of this bridge consists of series resonance ckt.

the series resonance circuit is formed by R_d , C_d & L_d series, all the other arms consists of resistors only.
using the equation for balance $Z_1 Z_4 = Z_2 Z_3$

$$Z_1 = R_b, Z_2 = R_c; Z_3 = R_a \text{ & } Z_4 = R_d + j\omega L_d - j/\omega C_d$$

$$\therefore R_b (R_d + j\omega L_d - \frac{j}{\omega C_d}) = R_a R_c$$



Counting real and imaginary term

$$R_b R_d = R_a R_c \quad \& \quad \delta w_B = \delta w_C$$

$$R_d = \frac{R_a R_c}{R_b} ; \quad \omega^2 = \frac{1}{L_d C_d}$$

$$\therefore f = \frac{1}{2\pi\sqrt{L_d C_d}}$$

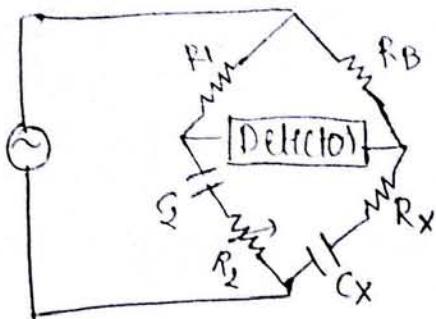
The bridge can be used to measure unknown inductance (C_d). The losses R_d can be determined by keeping a fixed ratio $\frac{R_a}{R_b}$ and using a standard variable resistance to obtain balance.

- If an inductance is measured a standard capacitor is varied until balance is obtained.
- If a capacitance is being measured a standard inductor is varied until balance is obtained.

The operating frequency of generator must be known in order to calculate unknown quantity. Balance is indicated by minimisation of sound in the headphones.

* SIMILAR ANGLE BRIDGE:

This bridge is known as similar angle bridge because only capacitive elements are involved. It is also known as capacitance comparison bridge.



At balance

$$R_1(R_x - j\omega C_x) = R_3(R_2 - j\omega C_2)$$

$$R_1 R_x - R_1 j\omega C_x = R_2 R_3 - R_3 j\omega C_2$$

$$R_1 R_x = R_3 R_2$$

$$\therefore R_x = \left(\frac{R_1}{R_2}\right) R_3$$

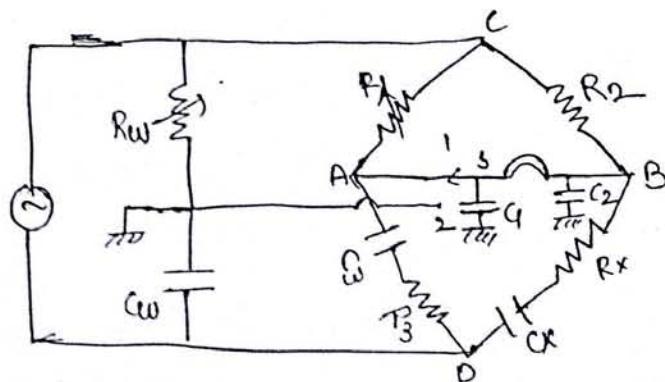
$$R_1 C_x = R_3 C_2$$

$$C_x = \frac{R_3 C_2}{R_1}$$

$$C_x = \left(\frac{R_3}{R_1}\right) C_2$$

The expression for R_x & C_x does not contain the term or E. Hence the measurement of the unknown elem is independent of the supply frequency or the supf voltage.

WAGNER'S EARTH (GROUND) CONNECTION:-





- When performing measurements at high frequency, stray capacitance b/w the various bridge elements and ground and between the bridge arms themselves becomes significant.
- This introduces an error in the measurement, when small values of capacitance and large values of inductance are measured.
- An effective method of controlling these capacitances, is to enclose the elements by a shield and to ground them. It does not eliminate capacitance but makes it constant in values.
- Another effective and popular method of eliminating these stray capacitances is to use a wagner's ground connection. C_1 & C_2 are the stray capacitances.
- R_w & C_w form the potential divider. The junction of 'R_w' & C_w is grounded and is called wagner's ground connection.
- The detector is connected to point 1 and R_i is adjusted for null (or) min sound in the head phones. The switch is then connected to point 2 which connects the detector to wagner's ground point. Resistor 'R' is now adjusted for min sound. When the switch is connected to point 1 again there will be some imbalance. Resistors R_i & R



are then adjusted for min sound and this procedure is repeated until a null is obtained on both switch pair 1 & 2. This is the ground potential.

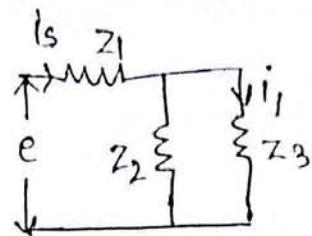
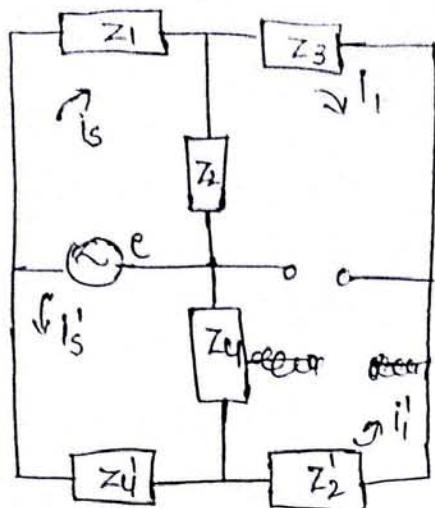
- The stray capacitances C_1 & C_2 are effectively short circuit and have no effect on the normal bridge balance. The capacitance from point C10D to GND are also eliminated by the addition of wagners ground connection, since current through these capacitors enter wagners ground connection.
- The addition of the wagners ground connection does not affect the balance condition, since the procedure for measurement remain unaltered.

TWIN-T NULL NETWORK:-

Also known as parallel T-NW, it will have two di T-Networks arranged in parallel with ILP & OLP turns

- A zero olp is obtained when the ckt impedance of the individual branches are so arranged that the transmission through the two-T-null nw to the olp turn is equal in magnitude but opposite in phase this is the condition for balance.

Twin-T nlw



simplified form of one T-r

features of twin T-nlw are as follows.

- 1) The ilp and olp terminals are in parallel, in other CKTs seen so far they are in perpendicular direction.
- 2) The ilp & olp have a common terminal that can be grounded, this minimises the shielding problems. Due to common ground terminal a shielding transformer is not required (in the case of wheat stone bridge).
- 3) Because of the lead-lag nlw is used in a twin-T nlw balancing is obtained easily.
- 3) Because of the lead-lag nlw is used in a twin-T nlw balancing is obtained easily.

If a balance is to be obtained

$$i_1 + i_1' = 0 \quad (\text{or})$$

$$i_1' = \frac{e}{z_1 + z_3 + \left(\frac{z_1 z_3}{z_2} \right)} = \frac{-e}{z_1^2 + z_2 + \frac{z_1 z_3}{z_2}}$$



Cross multiplying and bringing like terms to RHS

$$e \left[z_1 + z_2 + \frac{z_1 z_3}{z_2} + z_1 z_3 + z_3 + \frac{z_1 z_3}{z_2} \right] = 0 \quad (04)$$

$$z_1 + z_3 + \frac{z_1 z_3}{z_2} + z_1 + z_2 + \frac{z_1 z_3}{z_2} = 0$$

This is the condition for balance in a twin-T n/w. Similar to other AC bridges, this bridge can also be balanced by changing the supply frequency 'f' of the AC source, in addition to changing the supply voltage (or) component values such as R , L & C .

- However, both the phase angle and magnitude conditions must be satisfied for balance.

This is the

- In Twin-T n/w analysis assume o/p is shorted and determine the condition for which the short circuit current passes through the two T-n/w equal in magnitude but opposite in phase.

Let i_1 be the o/p current for the T n/w, z_1, z_2 & z_3 . z_2 & z_3 are parallel because o/p is assumed short circuit. The parallel combination of z_2 & z_3 is in series with z_1 .

$$z_1 = z_s = z_1 + \frac{z_2 z_3}{z_2 + z_3}$$

$$\therefore i_1 = \frac{e}{z_1 + z_2 + \frac{z_1 z_3}{z_2}}$$

$$\frac{z_1 z_2 + z_1 z_3 + z_2 z_3}{z_2 + z_3}$$

$$\frac{z_2 (z_1 + z_3 + \frac{z_1 z_3}{z_2})}{z_2 + z_3}$$

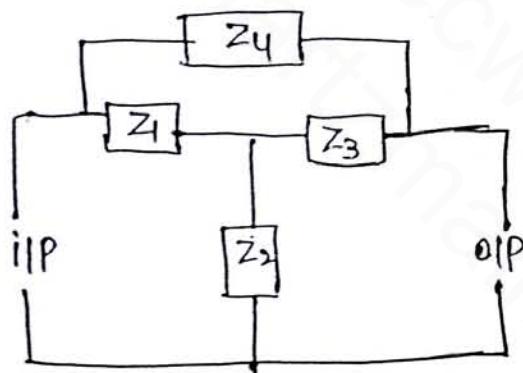
similarly the expression for i_1 through the second T-nlw is

$$i_1 = \frac{e}{z_1^1 + z_3^1 (z_1^1 z_3^1 / z_2^1)}$$

$$i_S = \frac{e}{z_1 + \frac{z_2 z_3}{z_2 + z_3}}$$

$$i_1 = \frac{i_S z_2}{z_2 + z_3} \quad (i_S = e/z_S)$$

BIDGE T-NLW:-



It is simplified form of bridge T-nlw, it has less no of components compared to twin-T nlw, but it is less accurate compared to twin-T nlw.

- This ckt is used to determine the incremental va of inductances and L_{eq} of radio frequency coils.
- The main advantage of this ckt is that shielding problems are less, and shielding transformer is not required.
- compared to twin-T nlw $z_2^1 = 0 \cdot c$ and $(z_2^1 + z_3^1) =$

in bridge-T N/W therefore the equation is

$$e \left(z_1' + z_2' + \frac{z_1' z_3'}{z_2} + z_1 + z_3 + \frac{z_1 z_3}{z_3} \right) = 0$$

can be written as for the balance equation for the bridge T-N/W

$$\therefore z_2' = 0 \text{ C} \quad \text{&} \quad \frac{z_1' z_3'}{z_2'} = 0$$

$$z_1' + z_3' = z_4$$

Therefore the equation of twin-T N/W can be modified and gives the equation at balance for the bridge-N/W as

$$\boxed{z_1 + z_3 + \frac{z_1 z_3}{z_2} = z_4}$$

Detectors:

- Telephone Receivers are connected through a transformer for impedance matching at 250-500Hz range.
- For AC bridges telephone receivers are commonly used as detectors.
- At lower and higher audio frequencies (where the ear is not sensitive) a cathode ray tube (or) a CRO can be used.
- A tuned amplifier with an indicating device can also be used as a detector.





- At radio frequencies ordinary radio receivers can be employed as detectors.
- The impedance of head phones can be in the range few hundred ohms to 5k Ω .
- The oscillatory input power required depends upon indicator and generally it is in the range of 50-200 mW.
- AC voltmeters and ammeters can also be used as detectors if they have the required sensitivity.